Perturbative analysis on infrared aspects of noncommutative QED on R^4

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Abstract

Here we examine the noncommutative counterpart of QED, which is called as noncommutative QED. The theory is obtained by examining the consistent minimal coupling to noncommutative U(1) gauge field. The $*$-product admits the coupling of the matter with only three varieties of charges, i.e., 0, +1 and -1. The ultraviolet divergence of noncommutative QED can be absorbed by redefinition of the theory at one loop level. To examine the infrared aspect of the theory the anomalous magnetic moment is calculated. Its depedence on the direction of photon momentum reflects the Lorentz symmetry violation of the system. The explicit calculation of the finite part of the vacuum polarization shows the presence of hard infrared singularity like $1/(q \cdot C^T C \cdot q)$ ($C^{\mu\nu}$ is a noncommutative parameter.) which also exists in noncommutative Yang-Mills theory. It might indicate the potential between the static charges dumps promptly faster than in ordinary QED. We also consider the extension to chiral gauge theory in the present context, but the requirement of anomaly cancellation allows only noncommutative QED.

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1 Introduction

Noncommutative Yang-Mills theory attracts one's interest after it emerged in the BPS solution of Matrix theory [1, 2], and is realized as the effective theory around one of the perturbative vacua of superstring theory with constant B background [3]. Prior to such recent development noncommutative geometry and the construction of field theory has been developed [4] to describe a world with a fuzzy structure by deforming the algebra of functions. Although the noncommutative geometry appearing in the context of perturbative superstring theory does not seem to have connection with quantum gravity, the general noncommutative geometry may accomodate the essential ingredients of quantum geometry (which we do not know at present) by its new machienary. Assuming the latter fact we are naturally inclined to investigate the quantum mechanical aspect of the theory defined on the noncommutative geometry in order to argue whether it really reflects the miscroscopic structure of our world. But the first thing to

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do is to analyze the simplest toy model and know that the theory is well defined even in the ultraviolet region. It is the most important subject to know how the infrared physics is affected, and whether it provides more desirable fundation for contructing the concrete model.

Since it appears in string theory, noncommutative Yang-Mills (NCYM) theory is widely discussed. The realization of such a theory on the set of the ordinal functions modifies the product of two functions in terms of "stared" (mentionded hereafter as "*"-) product. Here we consider how the matter field couples to Yang-Mills field in such a manner that the theory respects this rule of product as well as the gauge symmetry. The simplest such a candiate would be a noncommutative counterpart of QED, in which electron with a definite charge is present. As will be shown in Sec. 2 inclusion of matters does not have so many varieties. In spite of the base manifold considered here being topologically trivial, charge is limited to three varieties; 0, 1 and -1, the precise meaning of which is defined in Sec. 2. This is stronger requirement than the charge quantization on compact manifold such as torus. We expect that there would be Hilbert space realization for the theory with the fields of charge $\pm$ 1 on noncommutative goemetry and a mapping rule to the function space on the usual coordinates as was found in NCYM theory. Thus we call the fields with charge $+1$ as "electron" and the theory including such a object and "photon" as noncommutative QED (NC-QED) in this paper. Note that the massive scalar fields receive quadratic divergence which leads us to lose control of ultraviolet (UV) divergence unless supersymmetry forbids its appearance. Contrarily the usual fermion mass receives only logarithmic divergence in four-dimensional QED or QCD. Nonlocal generalization of the interactions is then expected not to drastically modify the divergent structure as experienced in NCYM system, which also needs further clarification. Thus here the system involving fermions with charges ± 1 is examined primarily.

Although NCYM system is related to ordinary Yang-Mills system by rather complex map [5], there would be nontrivial quantum mechanical dynamics in NCYM without maximal supersymmetry. One aspect of NCYM theory has been arugued to describe the large N ordinary Yang-Mills theory with a fixed t'Hooft coupling constant in the high momentum region [6]. It relies on the diagramatic expansion and the pattern of momentum-dependent phase factors appearing in the theories. More direct connection through the correlators of the gauge invariant operators is welcome in spite of curious nature of general Wilson loop operators in noncommutative Yang-Mills theory [6, 7, 8]. But it is beyond our present scope.

In order to access to the infrared side, the first thing we can do is to investigate the perturbative aspects. The perturbative analysis on UV structure of NCYM theory has been done in Ref. [9, 10]. The infrared side on NCYM as well as NC-QED is our primary concern. Naive continuation of asymptoticcally free nature [9] indicates the existence of such a dynamical scale as $\Lambda_{\rm QCD}$ at which the coupling constant diverges while naive commutative limit reduces to abelian gauge theory. The renormalization refers to the structures much higher than noncommutative energy scale, so it is natural that the commutative limit of the renormalized theory do not reduce it back to its commutative counterpart. We would like to begin in this paper with seeking the dynamics which could not be reached in local field theory, by first examining a simple model, NC-QED.

The paper is organized as follows: Sec. 2 is conerned with construction of classical action involving the matter fields and showing that the allowed choice is quite limited. In Sec. 3 noncommutative QED theory is quantized and the anomalous magnetic dipole moment is calcualted to see whether the radiative effect from noncommutative extension appears in the finite quantities. There the infrared behavior is further investigated by observing the finite part of the vacuum polaization of photon. There a hard infrared singularity is found. Such a singularity also exists in NCYM theory. It suggests "duality" between the infrared and UV sides of the theory. The extension to the chiral gauge theory is also examined in Sec.4, but it is found that there is *no* chiral gauge theory. Sec. 5 is devoted to the discussion and conclusion of the present paper.

2 Noncommutative QED

Pure noncommutative U(1) Yang-Mills action

$$S_{YM} = \int d^d x \left(-\frac{1}{4g^2}\right) F_{\mu\nu} * F^{\mu\nu} \,, \tag{1}$$

(Space-time dimension d is set equal to four in final.) is nothing but the one obtained from the ordinary SU(N) Yang-Mills action by replacing the matrix multiplication to the "star" (hereafter referred as $*$-)product

$$A * B(x) \equiv e^{\frac{1}{2i}C^{\mu\nu}\partial_\mu^{(\xi)}\partial_\nu^{(\eta)}} A(x+\xi)\, B(x+\eta)\Big|_{\xi,\eta\to 0} \,, \tag{2}$$

with an antisymmetric matrix $C^{\mu\nu}$ which characterizes noncommutativity of space-time by modifying the algebra of functions. Even in U(1) case A_μ couples to itself since the field strength $F_{\mu\nu}$ of A_μ has the nonlinear term

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu,\, A_\nu]_{\mathrm{M}} \,, \tag{3}$$

where $[A,\, B]_{\mathrm{M}}$ denotes Moyal bracket:

$$[A,\, B]_{\mathrm{M}} = A * B - B * A \,. \tag{4}$$

The $*$-product obeys the associative law which is also satisfied by the matrix algebra so that the manipulation in this case resembles the one experienced in the calculus of matrix. Thus it is quite simple to see the action (1) is invariant under

$$A_\mu(x) \to A'(x) = U(x) * A_\mu(x) * U^{-1}(x) + iU(x) * \partial_\mu U^{-1}(x) \,, \tag{5}$$

where $U(x) = (e^{i\theta(x)})_*$ is defined by an infinite series expansion of multiple $*$-product of scalar function $\theta(x)$. $U^{-1}(x) = (e^{-i\theta(x)})_*$ is similarly defined and plays the role of the inverse of $U(x)$. Here and hereafter we assume that the fields decrease so promptly at infinity that the space-time integral of a Moyal bracket (which corresponds to the trace of the commutator in the "matrix language") vanishes.
The coupling of "electron" to gauge field in noncommutative U(1) Yang-Mills theory receives a severe restriction. The gauge transformation (5) for the gauge field shows that the simple candidate of the interaction $\psi(x) * A_\mu(x)$ or $A_\mu(x) * \hat{\psi}$ implies that ψ and $\hat{\psi}$ must transform as

$$\psi(x) \to \psi'(x) = U(x) * \psi(x) \,, \quad \hat{\psi}(x) \to \hat{\psi}'(x) = \hat{\psi}(x) * U(x)^{-1} \,, \tag{6}$$

respectively in order for each product with gauge field to transform in the same way as the original field. The covariant derivative

$$D_\mu\psi = \partial_\mu\psi - iA_\mu * \psi, \quad D_\mu\hat{\psi} = \partial_\mu\hat{\psi} + i\hat{\psi} * A_\mu \,, \tag{7}$$

also transforms covariantly in the same way as the original objects. Since the commutative limit leads to the fields with +1 charge and −1 charge respectively in ordinary QED, so we call the field ψ in the above a field with +1 charge and referred hereafter as "electron" (opposite to the usual convention). Then the action

$$S_{\text{matter}} = \int d^d x \left(\bar{\psi} * \gamma^\mu i D_\mu \psi - m\bar{\psi} * \psi \right) , \tag{8}$$

is invariant under local U(1) symmetry since $\bar{\psi}$ [1] behaves in the same manner as $\hat{\psi}$. The field with charge +1(−1) in noncommutative case would correspond to (anti-)fundamental representation in ordinary nonabelian gauge theory. It is also reminiscent of such features that noncommutative gauge theory carries the internal degrees of freedom by imbedding them into the space-time geometry itself. This is the reverse process of the reduction of the space-time degrees of freedom into the internal ones in the large N gauge theory [11]. When we pursue this correspondence further, we are inclined to guess that the higher-rank representation of SU(N) gauge theory may convert into some matter fields in noncommutative gauge theory. It would be the counterpart of the fields of integral multiple of unit charge from the view point of noncommutative generalization of U(1) gauge theory. Actually the adjoint representation corresponds to a field $\chi(x)$ with zero charge in total but transforming in the by-product form

$$\chi(x) \to \chi'(x) = U(x) * \chi(x) * U^{-1}(x) . \tag{9}$$

Its covariant derivative is given by Moyal bracket. However we cannot find the counterpart of the second-rank antisymmetric representation, etc, of SU(N) gauge theory. The $*$-product admits only the fields with charge 0, +1 or −1. As a by-product the vacuum expectation value of Wilson loop operator for a rectangular loop becomes associated with the ground state energy acting between two sources of charges, +1 and −1, as usual.

3 Perturbative Aspects in Infrared Side

We are interested in the quantum mechanical aspect of the theory defined by the sum of (1) and (8)

$$S_{\text{NC-QED}} = \int d^d x \left(-\frac{1}{4g^2} F_{\mu\nu} * F^{\mu\nu} + \bar{\psi} * \gamma^\mu i D_\mu \psi - m\bar{\psi} * \psi \right) . \tag{10}$$

As explicit computation shows, the integral over the loop momentum can be defined only in the restricted kinematical region if the component of $C^{\mu\nu}$ on a plane including the time direction is nonzero. This would have the common origin of the critical electric field of string theory with electric field background [12]. Thus we consider the theory with nonzero C^{23} but zero C^{01} in the canonical basis for antisymmetric matrix $C^{\mu\nu}$.
Perturbation theory begins with rescaling $A_\mu \to g A_\mu$ and gauge fixing. BRST quantization as in ordinary QCD theory leads the gauge fixing and Faddeev-Popov (FP) terms

$$S_{\text{GF}} = \int d^d x \left(-\frac{1}{2\alpha} \partial_\mu A^\mu * \partial_\nu A^\nu + \frac{1}{2} \left(i\bar{c} * \partial^\mu D_\mu c - i \partial^\mu D_\mu c * \bar{c} \right) \right) . \tag{11}$$

[1] To compute the form factor of the on-shell electron coupling to photon we consider a theory in Minkowski space. Thus $\bar{\psi} = \psi^\dagger \gamma^0$.

Quantization is defined by perturbative expansion due to Feynman rule as was done for NCYM theory in Ref. [9], but now derive from the actions (10) and (11)

The extra ultraviolet divergent contributions arise in addition to those already appearing in NCYM theory. As we concentrate on reporting on the infrared phenomena in this short article, we postpone to describe the detail about ultraviolet divergence at one-loop level to the future extended volume [13], and state the results only in brief: All the one loop ultraviolet divergence can be subtracted by the local counterterms with maintaining the equalities among various Z factors (wave function renormalization constant, etc.) required from gauge invariance. The β function becomes for N_F number of copies of electron fields

$$\beta(g) = \frac{1}{g} Q \frac{dg}{dQ} = -\left(\frac{22}{3} - \frac{4}{3} N_F\right) \frac{g^2}{16\pi^2} \,. \tag{12}$$

A contribution $\frac{22}{3}$ is due to the structure similar to nonabelian dynamics, SU(2) Yang-Mills theory [9]. However the matter contribution is that found in ordinary QED theory with unit charge, *not* that of the quarks belonging to the fundamental representation of SU(2) gauge theory (in which $\frac{2}{3}$ instead of $\frac{4}{3}$ per flavor found in (12)).

In this analysis there is an important feature that should be kept in mind for the analysis of infrared aspect of the theory. Only *planar* diagrams can have overall divergence. Noncommutativity of the theory manifests itself in the form of a momentum-dependent phase associated with the vertex in Feynman rule. "Planar" diagram is a portion of the contributions which has a definite phase factor, but it only depends on the external momenta, not on any loop momenta. Once loop momentum enters in the phase factor, the suppression factor which depend on the external momentum through $C^{\mu\nu}$ is always induced. This is the same feature already shared by NCYM theory [14, 6].

To observe an infrared aspects of the theory, the leading correction to magnetic dipole coupling is estimated. The extraction of dipole coupling in the $\psi\bar{\psi}A_\mu$ vertex function yields

$$\begin{aligned} i\,\Gamma^\mu(p_I, p_F, q)|_{\text{dipole}} &= ig^3 \Big[e^{\frac{i}{2} p_I \cdot C \cdot p_F} H(1,p,q) \\ &\quad + e^{\frac{i}{2} p_I \cdot C \cdot p_F} H(0,p,q) - e^{-\frac{i}{2} p_I \cdot C \cdot p_F} H(1,p,q) \Big] \, m i \sigma^{\mu\nu} q_\nu \,, \end{aligned} \tag{13}$$

where q is the incoming photon momentum, and p is connected to the incoming electron momentum p_I and the outgoing electron momentum p_F through

$$p_I = p - \frac{q}{2}, \quad p_F = p + \frac{q}{2} \,. \tag{14}$$

The matrix $\sigma^{\mu\nu}$ is here $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$. The function $H(\eta, p, q)$ appearing in (13) is

$$\begin{aligned} H(\eta, p, q) &= \int_0^\infty i d\alpha_0 \int_0^\infty i d\alpha_+ \int_0^\infty i d\alpha_+ \frac{1}{[4\pi(\alpha_0 + \alpha_+ + \alpha_-)i]^2} \\ &\quad \times 2 \left(\frac{\alpha_+ + \alpha_-}{\alpha_0 + \alpha_+ + \alpha_-} - \left(\frac{\alpha_+ + \alpha_-}{\alpha_0 + \alpha_+ + \alpha_-} \right)^2 \right) \\ &\quad \times \exp\left[-i \frac{1}{\beta} \Big\{ (\alpha_+ + \alpha_-)^2 m^2 + \alpha_+ \alpha_- (-q^2) \right. \\ &\qquad\qquad \left. - \eta(\alpha_+ + \alpha_-)(p \cdot \tilde{q}) + \eta^2 \frac{\tilde{q}^2}{4} \Big\} \right] \,, \end{aligned} \tag{15}$$

where $\tilde{q}^\mu = C^{\mu\nu} q_\nu$ has the dimension of length. $H(0,p,q)$ becomes $\frac{1}{8\pi^2 m^2}$ for on-shell photon. $H(1,p,q)$ can be written in terms of a modified Bessel function of the second kind $K_1(x)$ [15]

$$H(1,p,q) \;=\; \frac{1}{8\pi^2}\int_0^1 d\alpha_+ \int_0^{(1-\alpha_+)} \frac{(\alpha_+ + \alpha_-) - (\alpha_+ + \alpha_-)^2}{(\alpha_+ + \alpha_-)^2 m^2 + \alpha_+\alpha_-(-q^2)}\, x\, K_1(x)\,, \tag{16}$$

where $x = (-\tilde{q}^2)\left\{(\alpha_+ + \alpha_-)^2 m^2 + \alpha_+\alpha_-(-q^2)\right\}$. Since $K_1(x) \sim \frac{1}{x}$ for $x \sim 0$ we can take q^2 and $\tilde{q}^2$ [2] to zero without confronting with any singularities. Thus for $q^2 = 0$ and $\tilde{q}^2 = 0$, $H(1,p,q)$ is equal to $H(0,p,q)$. Therefore the leading correction to the magnetic dipole moment is the same for ordinary QED and NC-QED. But the non-zero photon momentum in the direction transverse to $(2,3)$ plane is allowed. Eq. (13) shows that the strength of magnetic dipole coupling is affected for such a photon in general.

We examine the infrared behavior of the renormalized vertex functions, especially the vacuum polarization for photon. As the analysis is lacking even for NCYM theory [3], the common contributions, the FP-ghost loop, gauge boson loop are examined here. Taking Feynman gauge for simplicity, they can be written in terms of Schwinger parameterization [17]

$$\begin{aligned}
i\Pi^{\mu\nu}_{\text{ghost}+33}(q) = ig^2 &\int_0^\infty i d\alpha_+ \int_0^\infty i d\alpha_- \frac{1}{(4\pi\beta i)^{d/2}} \exp\left[-i\frac{\alpha_+\alpha_-}{\beta}(-q^2)\right] \\
\times &\left[\left(1 - \exp\left[-i\frac{1}{\beta}\frac{\tilde{q}^2}{4}\right]\right) \times \left\{g^{\mu\nu}\left((3d-4)i\frac{1}{\beta} + \left(5 - 2\frac{\alpha_+\alpha_-}{\beta^2}\right)q^2\right)\right.\right. \\
&\qquad\qquad\left. + q^\mu q^\nu \left((d-6) - 4(d-2)\frac{\alpha_+\alpha_-}{\beta^2}\right)\right\} \\
&\left. + \exp\left[-i\frac{1}{\beta}\frac{\tilde{q}^2}{4}\right] \times \frac{1}{\beta^2} \times \left\{-\frac{1}{2} g^{\mu\nu}\tilde{q}^2 + (2-d)\tilde{q}^\mu\tilde{q}^\nu\right\}\right]\,, \\
i\Pi^{\mu\nu}_4(q) = 2(d-1) i g^2 g^{\mu\nu} &\int_0^\infty i d\alpha \frac{1}{[4\pi\alpha i]^{d/2}} \left(1 - \exp\left[-i\frac{1}{\alpha}\frac{\tilde{q}^2}{4}\right]\right)\,.
\end{aligned} \tag{17}$$

where $\beta = \alpha_+ + \alpha_-$. The first quantity in eq. (17) is the contributions from the ghost loop and the gluon loop induced through the two trilinear gauge couplings. The other quantity is due to the quartic self-interaction of gauge boson. The evaluation is similar as found in Ref. [17]. The exponential factor $\exp[-i/(4\beta\tilde{q}^2)]$ acts as the cutoff for the ultraviolet divergence. The latter quantity in eq. (17) is calculated as:

$$i\Pi^{\mu\nu}_4(q) = i\frac{g^2}{16\pi^2} g^{\mu\nu} \frac{-24}{-\tilde{q}^2}\,, \tag{18}$$

containing a hard singularity $1/\tilde{q}^2$. It would be cancelled by the term from $\Pi^{\mu\nu}_{\text{ghost}+33}(q)$. To evaluate it we need to perform the integrals

$$\int_0^\infty \frac{d\rho}{\rho^n} \exp\left(-\rho - \frac{1}{\rho}\frac{x^2}{4}\right) = (-\frac{x}{2})^{-n} \frac{d^n}{dx^n}[x K_1(x)]\,, \tag{19}$$

[2]Since we consider the situation that only C^{23} is nonzero, thus there is an on-shell photon with a finite spatial momentum.

[3]See also Ref. [16].

where x^2 is proportional to $\tilde{q}^2 q^2$ in the present context. Using the asymptotic behavior of $K_1(x)$ around $x \sim 0$ available in a mathematical literature [15] we can derive the useful formula, by setting $a = x/2$,

$$
\begin{aligned}
\int_0^\infty \frac{d\rho}{\rho} \exp\left(-\rho - \frac{1}{\rho}\frac{x^2}{4}\right) &= -\ln(a^2)\left(1 + \mathcal{O}(a^2)\right) - 2\gamma_E + \mathcal{O}(a^2)\,, \\
\int_0^\infty \frac{d\rho}{\rho^2} \exp\left(-\rho - \frac{1}{\rho}\frac{x^2}{4}\right) &= \ln(a^2)\left(\frac{1}{2}\frac{1}{a^2} + \frac{3}{2} + \mathcal{O}(a^2)\right) \\
&\quad + (\gamma_E + 1)\,\frac{1}{a^2} + \left(6\gamma_E - \frac{23}{4}\right) + \mathcal{O}(a^2)\,, \\
\int_0^\infty \frac{d\rho}{\rho^3} \exp\left(-\rho - \frac{1}{\rho}\frac{x^2}{4}\right) &= -\ln(a^2)\left(\frac{3}{2}\frac{1}{a^2} + \frac{5}{4} + \mathcal{O}(a^2)\right) - \frac{1}{2}\frac{1}{a^4} \\
&\quad + \left(-6\gamma_E + \frac{17}{4}\right)\frac{1}{a^2} + \left(-30\gamma_E + \frac{1117}{24}\right) + \mathcal{O}(a^2)\,, \qquad (20)
\end{aligned}
$$

where γ_E is Euler constant. They allow us to compute the singularity of $\Pi^{\mu\nu}_{\text{ghost}+33}(q)$ for small $\tilde{q}^2$. The leading singularity is found as

$$
i\Pi^{\mu\nu}_{\text{ghost}+33}(q) \sim i\frac{g^2}{16\pi^2} g^{\mu\nu} \frac{1}{-\tilde{q}^2}\left(32\gamma_E + 36\right)\,. \qquad (21)
$$

Therefore the sum of (18) and (22) does not vanish:

$$
i\Pi^{\mu\nu}(q) \sim i\frac{g^2}{16\pi^2} g^{\mu\nu} \frac{1}{-\tilde{q}^2}\left(32\gamma_E + 12\right)\,. \qquad (22)
$$

At first glance such a singularity ($|x-y|_T^2$ in coordinate space where the norm siginified by a subscript T is the length along (2,3)-plane.) conflicts with the Slavnov-Taylor identity naively derived from BRST symmetry of the system. Thus we do not convince ourselves that such a singularity is real or an artifact of the dimensional regularization which might be insufficient to pertain the gauge symmetry in the infrared region. Before proceeding further analysis on NC-QED, this point needs to be clarified.

4 Chiral Gauge Theory

Until now all the fermions are assumed to be Dirac fermions. It is naturally tempted to pursue the extension to chiral gauge theory. The classical analysis given in Sec. 2 is irrelevant to the chiral property of fermion. Thus Weyl fermions can have the charge $+1$ or -1. The right-handed fermion with $+1$ charge is easily seen to be replaced by its CP conjugate (the left-handed) fermion also in the present context. The chiral gauge theory simply implies that the number of the left-handed fermions with $+1$ charge is not equal to one with -1. The question is whether such a theory can circumvent a triangle loop anomaly to define a consistent quantum theory or not.

It is quite an easy excise to see the triangle loop diagram is planar. The phase $e^{-\frac{i}{2}q_1 \cdot C \cdot q_2}$ depend only on the external photon momenta. Thus a triangle fermion loop diagram gives only planar contributions. Once we remind the correspondence between the current theory to ordinary nonabelian gauge system in which the external momentum plays the role of color in Yang-Mills theory, the remained integral is evaluated completely in the same manner as encountered in ordinary nonabelian gauge theory involving the fundamental and/or anti-fundamental Weyl fermions. From this observation, the number of the left-handed fermions with -1 charge has to match with the number of fermions with $+1$ in the system [4]. Such a theory is vector-like, i.e., noncommutative QED considered until the present sections.

5 Conclusion

In this paper we attempt to find the noncommutative analogue of QED and argue the perturbative aspects of its infrared dynamics. The anomalous magnetic dipole moment does not change at leading order for the photon moving in the direction along which noncommutativity is irrelevant. However the form factor seems to indicate the possible observation of Lorentz invariance SO(1,3) by controlling the direction of photon although the conventinal environment of measurement averages over the directions of photon. In order to discuss quantum aspects of the theory, it would be the best to calculate and investigate the radiatively corrected cross sections for Compton, or Mϕller scattering, or the photon-electron, photon-photon scattering processes with explicity specified helicities of the external states.
However before doing that we have to obtain a conclusive evidence of the hard singularity like $1/\tilde{q}^2$ encountered in the vacuum polarization of photon. Such a singularity exists also in NCYM theory and may reflect some "dual" role of $\tilde{q}$ and the original momentum q. Dyson resummation of such one particle irreducible contributions gives to the photon propagator [5]

$$\frac{ig^{\mu\nu}}{-q^2 + c_0 \frac{1}{-\tilde{q}^2}} = \frac{ig^{\mu\nu}(-\tilde{q}^2)}{c_0 + q^2 \tilde{q}^2} \,. \tag{23}$$

Thus the infrared side will also be in the week coupling regime as in the ultraviolet side (as long as β function in eq. (12) is negative). The form of the propagator indicates that some anomalous violation mechanism might occur. There would happen a crossover phenomena around noncommutative scale, and results in "duality" between the infrared and the UV regions.
The requirement of anomalous diagrams being cancelled in total is too strong for chiral gauge theory to emerge in the present context. It is an interesting and important subject to relax the requirement of such a complete cancellation.

Note added

During preparation of the paper, we find a preprint [16] reported a few days ago, which discussed the subject partly overlapping with the present paper. There it was concluded that the hard singularity like $1/\tilde{q}^2$ does not occur, contrarily to the present paper. As we do not know their detail computation, it needs some time to clarify such discrepancy, for instance, to analyse the Euclidean theory compactitified on torus. There will be a subtlety to argue the infrared limit

[4] We require that the triangular loop contribution cancels with each other for *all* momentum configuration. But it might be too strong requirement for noncommutative SU(N) gauge theory due to non-factorizability of color and phase factors, as suggested by Y. Kitazawa.

[5] The follwing argument is crude since it neglects the fact that there also appear logarithmic singularities about $-\tilde{q}^2$ in the vacuum polarization.

in the calculation of effective actions similar to the appearance of fictious holomorphic anomaly encountered in supersymmetric systems.

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